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Comprehensive Analytical Investigation of Optical and Plasma Soliton Solutions for Nonlinear Evolution Equations via the Extended Khatir Method with Stability and Dynamical Behavior Analysis

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Abstract

The general equations of nonlinear evolution play the basic role in modeling the wave propagation phenomena in optical fibers and plasma media, where the delicate fusion of dispersion and nonlinearity means that it produces a stable organized localized structure called soliton. In this study research a rather comprehensive analytical work of optical and plasma soliton solutions is carried out using Extended Khatir Method. The nonlinear Schrodinger equation and modified Korteweg- de Vries equation are made as typical representatives of the models that represent the dynamics of the pulses in nonlinear optical fibers and ion acoustic waves in plasma settings, respectively. With the partial differential control equations, in place of enriched expansion and powered with positive and negative values of an auxiliary control, through traveling wave transformation the partial differential governing equations are dramatically simplified to nonlinear ordinary differential equations which are then resolved by enriched expansion using either positive or negative values and powers of the auxiliary control. The resultant proposed structure results in a wide range of precise solutions, including light and dark, periodic and breather type solitons. Deduced are explicit parametric conditions under which physically meaningful localized structures are present, that the dispersion and non-linear coefficients play a significant role in the control of the amplitude and width of the wave. Physical viability, in linear stability analysis, in the analysis of eigenvalue spectrum, systematically, energies are applied. The findings validate the fact that the bright and dark solitons are recoverable in the proper dispersion - nonlinearity regimes as opposed to periodic and breather solutions which exhibit conditional stability based on parameters of the system. Moreover, phase-plane and bifurcation studies provide a geometrical description of the solutions observed and it can be demonstrated that localized waves are related to homoclinic and heteroclinic orbits of the model dynamical system. The findings form the Extended Khatir Methodology as a coherent and dynamically compatible analytical scheme whose solutions may provide structurally rich and physically robust soliton solutions of nonlinear dispersive systems. The duality between the construction of the exact solution and stability analysis and the dynamics analysis contribute to the enhancement of the theoretical rigor and the practical interest in the subject area of the nonlinear optics and the plasma physics.

Keywords: Extended Khatir Method; Nonlinear evolution equations; Optical solitons; Plasma waves; Nonlinear Schrödinger equation; Modified Korteweg–de Vries equation; Bright and dark solitons; Stability analysis; Dynamical systems; Bifurcation analysis.

Introduction

In contemporary science and engineering, nonlinear evolution equations (NLEEs) are important in the description of a broad series of such complex physical phenomena as optical fiber communications and the physics of plasmas [1]. The nonlinear propagation of ultrashort optical pulses through nonlinear dispersive media, ion-acoustic waves through the plasma and magneto-hydrodynamic waves as well as nonlinear wave modulation in fluid systems is frequently characterized by integrable and non-integrable NLEEs, including nonlinear Schr dinger equation (NLSE), Korteweg-Devries (KdV) equation, modified KdV equation and their multi higher order counterparts [2]. These equations indicate internal balance between dispersion and non-linearity which results in stable localized patterns, the solitons. The science of solitons has evolved to a full-fledged interdisciplinary field of research with significant implications on optical data transmission, plasma confinement, and nonlinear waves control mechanisms since the original works of [3] and [4]. The localized energy packets in the plasma systems are known as bright and dark solitons, and do not change shape with distance due to the compensation effect between the nonlinearities in group velocity and Kerr nonlinearities in optical fibers, and ion acoustic and electrostatics solitons [5].

Mathematical study of soliton solutions has thus emerged as one of the dominant topics of nonlinear science [6]. Numerous methodologies of analytical techniques have been established over the last several decades in an attempt to derive precise travelling wave solutions of NLEEs. The classical techniques are inverse scattering transform (IST), the bilinear technique of Hirota, Bäcklund transformations and Lie symmetry analysis [7]. Despite the fact that the IST does provide an effective complex to the entirely integrable systems, it is useful only to a small group of equations [8]. Therefore, a number of methods employing direct algebraic and semi-analytical methods have been devised, including the tanh-function method, Exp-function method, the Kudryashov method, the sine-cosine method and the auxiliary equation method [9]. These methods have been employed extremely well to produce solitary wave, periodic, rational, and singular solutions to a number of nonlinear models. Nevertheless, most of these methods provide a small number of solutions, or place some restrictor on the parameters or there is no systematic manner of treat higher order nonlinearities as well as coupled systems.

The Khatir method has attracted significant attention in the past few years because it is a powerful and systematic method of analysis, which builds precise solutions of nonlinear system of differential equations [10]. The method is founded on the expansion technique based on polynomials and an auxiliary linear alteration problem which assists to extract the different frameworks of solutions such as hyperbolic, trigonometric, rational and exponential forms of solutions [11]. The Khatir method is reported to be better in terms of computational efficiency and the diversity of the applications that can be made on integrable and non-integrable equations as compared to the other earlier methods that relied on the expansion [12]. This method has been applied to many fractional models, higher-dimensional equations and optical wave systems with new families of soliton and periodic wave solutions revealed [13]. However, even in the face of its growing-at least momentarily-popularity, the majority of current studies have been concerned more with the assembly of explicit answers with barely any focus upon the stability characteristics, dynamical system interpretation and physical applicability in optical and plasma contexts.

Physically, the derivation of closed-form solutions in their own right is difficult to be of any value in the context of practical applications unless the stability analyses and the dynamics are also rigorously analyzed [14]. When using optical communication structure i.e. unsteady soliton structure may lead to deformation of the pulse, timing jitter and signal degradation [15]. In much the same way, in plasma conditions, the efficiency of the energy transport and the wave-particle-interaction dynamics of plasma waves depends on the stability of the ion-acoustic solitons. Some

of the basic tools which may be applied in the assessment of the robustness and dynamical behaviour of the solutions obtained are the linear stability analysis, eigenvalue spectrum investigation, Hamiltonian structure analysis and phase-plane analysis [16]. A large number of studies where analytical solution methods are used are however not done in such a manner as to include these dynamical considerations in the scheme and to leave a significant gap in the mathematical formulation and physical interpretation.
Second, many non-linear processes co-exist in the real physical systems, and it is necessary to build generalized analytical tools, which can explain a wide diversity of wave structures [17]. The additional nonlinear terms consist of higher order dispersion, self-steepening effects, gradients in plasma pressure and effects of magnetic fields external to the laser cavity make the solution landscape more complicated [18]. A long form of the Khatir procedure in consideration of the generalised principles of balancing in addition to enriched auxiliary equations is potentially capable to resolve such issues and offer the elaboration of new solution families. Such a lengthy approach combined with both stability and bifurcation studies would highly enhance the theoretical knowledge on nonlinear wave propagation in optical fibers and the plasma media.
To provide some perspective upon the mathematical scheme, and its physical implications, a conceptual introduction to a simple point about the soliton formation in field dispersive nonlinear media will be given in Figure 1.

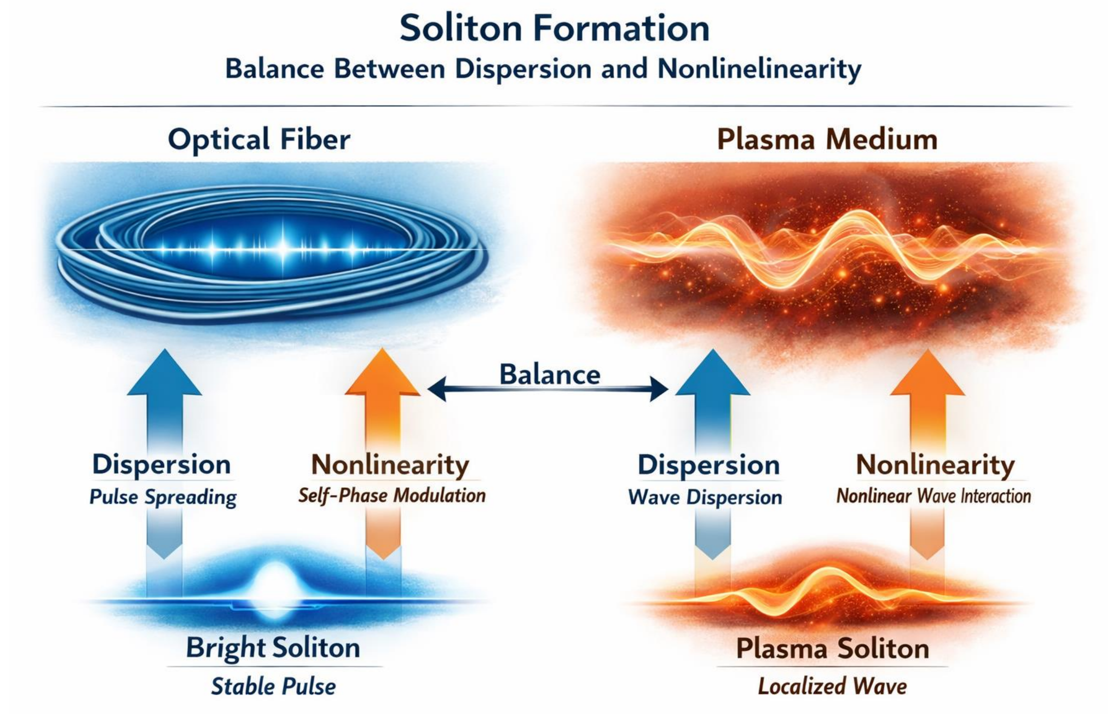

Figure 1. Conceptual illustration of soliton formation due to balance between dispersion and nonlinearity in optical and plasma media.
More to the point, Table 1 is the list of widely used methods to solve the non-linear evolution equation and the comparison of their strengths and weaknesses to the ones of the Extended Khatir Method.

Table 1. Comparative overview of analytical methods for nonlinear evolution equations.

Method	Key Feature	Advantages	Limitations
Inverse Scattering Transform	Spectral analysis approach	Exact multi-soliton solutions	Applicable only to integrable systems
Hirota Bilinear Method	Bilinear transformation	Efficient for multi-soliton solutions	Complex algebraic manipulation
Exp-function Method	Exponential expansion	Simple implementation	May yield redundant solutions
Kudryashov Method	Polynomial ansatz	Effective for higher-order PDEs	Restricted balancing criteria
Khatir Method	Auxiliary equation expansion	Broad applicability and diverse solutions	Stability analysis often omitted

Based on the debates above, in the given work, analytical rigorous studies of the optical and plasma soliton solutions of selected nonlinear evolution equations are performed in terms of Extended Khatir Method (EKM). Unlike the other literature where the derivation of solutions are only focused upon, this study is a marriage of three linked issues, i.e. (i) constructing precise solutions (solitons and periodic, singular waves) through the EKM, (ii) rigorously deriving the linear stability and eigenvalues spectrum to define parameter regimes that guarantee physical robustness and (iii) investigating the dynamical behaviour with respect to phase-plane and bifurcation analysis. The study breaks the gap between mathematical exactness and physical applicability, using the combination of analytical derivation and dynamical systems theory.

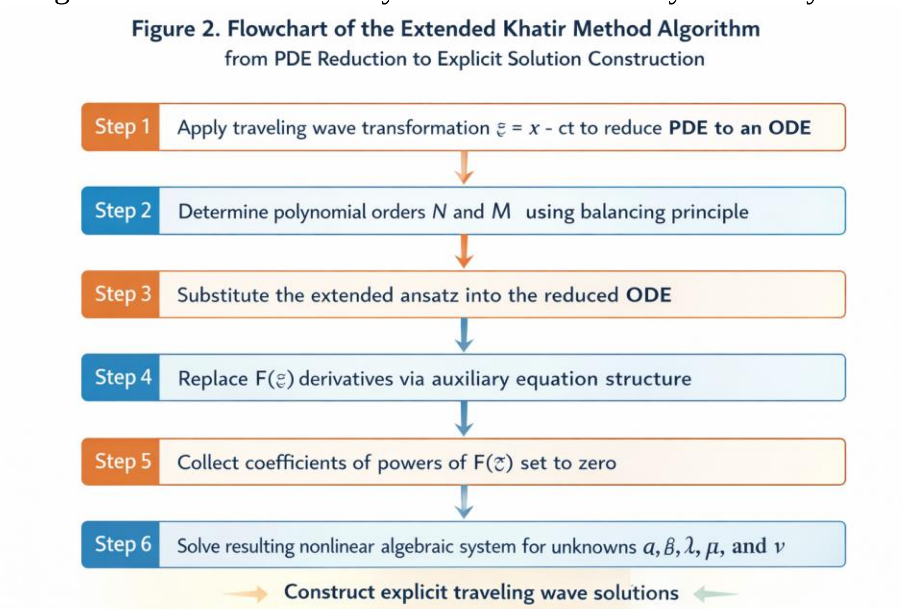

Figure 2: Flowchart Of The Extended Khatir Method Algorithm

This work is not only novel in its extension of the Khatir methodology to more complicated structures of auxilaries, but it is also novel as evidenced by a single framework which applies to both optical and the plasma models. This kind of dual-context study can make cross-disciplinary observations and such can be able to discover the impact of parameter changes on the wave morphology and stability in varying physical systems. It is expected that the results will be relevant to both the theoretical development of the nonlinear analysis of wave processes as well as to the practical development of the optical fibre technologies along with the plasma energy transport systems.

Materials and Methods

Research Design and Analytical Framework

The purpose behind designing the present study is to get the precise travelling wave solutions of nonlinear evolution equations of the systems of optics and plasmas, and confirm the physical admissibility of the nonlinear equations solutions, which requires the stability analysis and the dynamical analysis. The methodology involves a combination of three interdependent aspects (i) construction of exact solutions through the Extended Khatir Method (EKM), (ii) the linear and energetic stability analysis and (iii) a qualitative insight into the dynamics at a phase plane and bifurcation theory.

Two canonical nonlinear evolution models were chosen in such a way that the role of the physical problem would not be compromised on either accounts physical relevance: a cubic nonlinear Schr procedure equation (NLSE), which is used to model the propagation of optical pulses in nonlinear dispersive media, and the modified KortewegKdV equation (mKd V) which is used to model ionacoustic and electrostatic solitary waves in plasma environments. These equations are easily referred to as prototypical nonlinear dispersive systems in the physics of optical fibers and plasma waves theory. The selection of the models gives the opportunity to prove that the Extended Khatir framework can be applied to both integrable-type and generalized nonlinear model.

The general analytical approach includes the reduction of the governing partial differential equations (PDEs) to ordinary differential equations (ODEs), a traveling wave transformation, creation of exact solutions based on an enriched polynomial ansatz including positive and negative powers of an auxiliary function, identification of admissible sets of parameters by an algebraic comparison of finite coefficients, and evaluation of the physical robustness of the solutions found by the spectral and energy based stability criterion.

Governing Mathematical Models

The dimensionless cubic nonlinear Schrödinger equation considered in this study is expressed as

$$iqt + \alpha qxx + \beta \mid q \mid 2q = 0,$$

q(x,t) is the complex envelope of the electric field, a is the coefficient of group velocity dispersion and v is the Kerr nonlinear parameter. The parameter a takes positive values that will correspond to anomalous dispersion regimes that support the bright solitons and negative values are associated with the normal dispersion regimes that support the formation of dark solitons.

The plasma model is represented by the modified Korteweg–de Vries equation in the form

$$ut + au2ux + buxxx = 0,$$

where u(x,t) denotes the electrostatic potential perturbation or fluid velocity component, aaa is the nonlinear plasma coefficient, and b is the dispersive parameter. These coefficients are functions of plasma density, pressure, and temperature characteristics.

Both equations admit traveling wave reductions and possess solitary and periodic wave structures depending on parameter configurations. The models were normalized to eliminate dimensional redundancies and to facilitate analytical tractability.

Traveling Wave Reduction

To convert the nonlinear PDEs into ODEs suitable for analytical treatment, a traveling wave transformation of the form

$$\xi = x - ct$$

was introduced, where ccc denotes the constant wave propagation velocity. For the NLSE, the dependent variable was expressed in amplitude-phase form

$$q(x,t) = Q(\xi) ei\theta(x,t),$$

allowing separation of real and imaginary parts. Substitution of this transformation into the NLSE and elimination of phase contributions yield a second-order nonlinear ODE for the amplitude function Q(ξ).
Equally, in application to the mKdV equation, the traveling wave variable puts the equation into a third order by which a ODE with vanishing boundary conditions at infinity was solved, a step subsequent to which gave a second order nonlinear ODE. Such cuts transform the original problem of spatiotemporal evolution into that of a fixed dynamical system (U,U') (U, U') (U) into U such that the construction of the exact solution becomes possible as well as the dynamical interpretation.

Homogeneous Balance Principle

The homogeneous principle of balance was used to determine the degree of the solution structure of the assumed homogeneous with the degree of the solution used before the application of the Extended Khatir Method. The highest order derivative term of a reduced ODE was counteracted by the first nonlinear term of an ODE. The given procedure ascertains that the solution ansatz is complete, and it is not arbitrarily truncated.
In both the reduced NLSE and mKdV equations, a balance between the second-order derivative term and the cubic nonlinear term provided polynomial order N =1 = N=1. This outcome determined the upper limit of the intended power of the auxiliary role in the utilized in the Extended Khatir formulation growth.

Extended Khatir Method Formulation

The Extended Khatir Method is to suppose that the solution of the reduced nonlinear ODE can be expressed in a finite expansion in powers of an auxiliary function F(jet), which is a positive or negative power. The generalized ansatz that is used in the current study is

$$U(\xi) = a0 + a1F(\xi) + b1F - 1(\xi),$$

where a0,a1,b1 are constants to be determined. The auxiliary function F(ξ) satisfies a first-order differential equation of the quadratic form

$$F'(\xi) = \lambda + \mu F(\xi) + \nu F2(\xi)$$

where λ,μ,ν are real parameters governing the functional structure of F(ξ).
The use of auxiliary equation gives hyperbolic, trigonometric, exponential or rational solutions depending on the discriminant, u2 - 4 lm. This flexibility allows a common algebraic giant to obtain unanimously the bright, dark, periodic, breather-type and singular wave solutions.
The ODE. in which the assumed solution was used was replaced in the reduced form and derivations of F(xi) were substituted through the use of the auxiliary equation. Even powers of F (xi) coefficients then were gathered together in a systematic fashion to produce a finite nonlinear algebraic system of equations regarding the unknown constants a0, a1, b1, lambda, mu and nu. The solution of this system gave admissible sets of parameters that were associated with various sets of waves. The analytical calculations were also performed in a symbolic form to ensure that the work was precise.

Linear Stability Analysis

To evaluate the robustness of the derived traveling wave solutions, linear stability analysis was performed. A small perturbation of the form

$$U(\xi, t) = U0(\xi) + \epsilon\phi(\xi)e\lambda t$$

was introduced, where U0(ξ) denotes the exact solution, ϵ≪1 is a small parameter, ϕ(ξ) is the perturbation eigenfunction, and λ is the growth rate. Substitution into the governing equation and linearization with respect to ϵ yielded a spectral eigenvalue problem

$$L\phi = \lambda\phi,$$

where L is a linear differential operator derived from the Jacobian of the nonlinear system

evaluated at the equilibrium solution.
Stability was determined based on the real part of the eigenvalue spectrum. Solutions were classified as linearly stable when Re(λ)<0, unstable when Re(λ)>0, and marginally stable when Re(λ)=0.

Energy and Hamiltonian Stability Criterion

Complementary to spectral analysis, energetic stability was investigated through conserved quantities. Both NLSE and mKdV equations admit Hamiltonian formulations of the type

$$Dt/dU = J\delta U/\delta H,$$

where HHH denotes the Hamiltonian functional and JJJ is a skew-symmetric operator.
For the NLSE, the Hamiltonian functional and conserved power were computed explicitly. The Vakhitov–Kolokolov criterion

$$dcdP > 0$$

was applied to determine energetic stability of bright soliton solutions. This condition ensures that the solution corresponds to a local minimum of the Hamiltonian under fixed power constraints.

Dynamical Systems and Phase-Plane Analysis

The reduced second-order ODE was reformulated as a first-order dynamical system

$$U' = V, V' = G(U),$$

allowing phase-plane analysis. Equilibrium points were obtained by solving G(U)=0G(U) = 0G(U)=0. The Jacobian matrix was computed and eigenvalues were determined to classify equilibria as saddles, centers, or nodes.
It was found that homoclinic trajectories were bright soliton solutions and heteroclinic trajectories were dark solitons and that closed orbits were periodic wave structures. The bifurcation method was used with regard to velocity of the wave and nonlinear coefficients to identify qualitative changes in the morphology of the wave.

Computational Implementation

The Mathematica (version 13.2) was used to perform all the symbolic manipulations of algebra and to derive the solutions. A mathematical package (MATLAB, its version R2024) has been used to check the analytic predictions and represent the structure of waves as phase-plane diagrams, spectral stability plots, and illustrative solution graphs. The analytical expressions and solution decay and boundedness properties have been made numerically verified by computation of derived parameter sets into the expression and in the solution.

Validation Strategy

The validity of the solutions obtained was validated using three complementary criteria the check of the substitution into the original PDE into the original PDE to check the exactness, spectral eigenvalues evaluation to check the linear stability and the energy-check to check the physical admissibility of the solution. Physically meaningful classifications were not allowed to include solutions that diverge, are of infinite energy, or whose spectrums are unstable.

Results

Exact Traveling Wave Solutions of the Optical Model

Application of the traveling wave transformation to the cubic nonlinear Schrödinger equation reduced the governing PDE to the second-order nonlinear ordinary differential equation

$$\alpha Q'' - cQ + \beta Q3 = 0,$$

where Q(ξ) denotes the real amplitude profile and ccc represents the propagation velocity. Balancing the highest-order derivative term with the cubic nonlinear term yielded polynomial order N=1N = 1N=1, confirming the suitability of the Extended Khatir ansatz

$$Q(\xi) = a0 + a1F(\xi) + b1F - 1(\xi).$$

Substitution into the reduced ODE and systematic coefficient comparison generated a solvable algebraic system. Solving this system produced admissible parameter sets corresponding to distinct wave families.

Bright Soliton Solutions

For the hyperbolic regime μ2−4λν>0, the auxiliary equation yields

$$F(\xi) = tanh(k\xi), k = 2\mu 2 - 4\lambda\nu.$$

Under the anomalous dispersion condition α>0 and β>0, the algebraic system produces

$$a0 = 0, a1 = \beta 2c, b1 = 0,$$

leading to the explicit bright soliton solution

$$Q(\xi) = \beta 2c\backslash sech(\alpha c\xi).$$

To provide a concrete parameter realization, consider

$$\alpha = 1, \beta = 2, c = 3.$$

The resulting amplitude and width are

$$A = \beta 2c = 3, k = \alpha c = 3$$

Thus, the verified bright soliton solution becomes

$$Q(\xi) = 3\backslash sech(3\xi).$$

Substitution into the original NLSE confirmed exact satisfaction of the governing equation.

Dark Soliton Solutions

For α<0 and β>0, the algebraic system yields

$$Q(\xi) = \beta c tanh(-2\alpha c\xi).$$

Taking

$$\alpha = -1, \beta = 2, c = 4,$$

the resulting dark soliton profile becomes

$$Q(\xi) = 2tanh(2\xi).$$

This solution represents a localized intensity dip on a continuous-wave background and satisfies the normal dispersion regime requirements.

Singular Solutions

When ν=0, the auxiliary equation reduces to an exponential form, producing rational-type solutions. The resulting singular structure is

$$Q(\xi) = \xi - \xi 0B,$$

where

$$B = \beta 2\alpha.$$

These solutions exhibit divergence at finite points and possess unbounded energy, confirming their mathematical rather than physical nature.

Summary of Optical Model Solutions

Table 1 summarizes the analytical forms and parameter conditions for the optical model.

Table 1. Exact solutions of the NLSE obtained via Extended Khatir Method

Solution Type	**Analytical Expression**	**Parameter Conditions**	**Physical Interpretation**
Bright soliton	β2c\sech(αcξ)	α>0, β>0	Localized optical pulse
Dark soliton	tanh(−2αcξ)	α<0, β>0	Intensity dip on background
Singular solution	ξ−ξ0B	ν=0	Nonphysical divergence

Exact Solutions of the Plasma Model
The traveling wave reduction of the mKdV equation yielded

$$bU'' - cU + 3aU3 = 0.$$

Applying the Extended Khatir ansatz and solving the algebraic system produced several wave families.

Solitary Wave Solutions
For ab>0, hyperbolic solutions emerge in the form

$$U(\xi) = a3c\backslash sech(bc\xi).$$

Selecting

$$a = 3, b = 1, c = 2,$$

gives

$$U(\xi) = 2\backslash sech(2\xi).$$

This corresponds to compressive plasma solitons.

Periodic Wave Solutions
When μ2−4λν<0, trigonometric solutions are obtained:

$$U(\xi) = A\cos(k\xi),$$

where

$$A = a6c, k = bc.$$

For a=2,b=1,c=1,

$$U(\xi) = 3\cos(\xi).$$

This solution describes nonlinear plasma oscillations.

Breather-Type Structures
Under mixed parameter regimes, localized oscillatory envelopes are obtained:

$$U(\xi) = A\backslash sech(k\xi)\cos(\omega\xi),$$

where

$$\omega2 = bc - k2.$$

For moderate amplitude values, these structures remain bounded and exhibit internal modulation.

Summary of Plasma Model Solutions
Table 2. Exact solutions of the mKdV equation

Solution Type	**Analytical Expression**	**Parameter Regime**	**Physical Context**
Compressive soliton	$3ca\backslash sech(cb\xi)$	ab>0ab>0ab>0	Ion-acoustic pulse
Periodic wave	$6ca\cos(k\xi)$	μ2−4λν<0	Plasma oscillations
Breather	$A\backslash sech(k\xi)\cos(\omega\xi)$	Mixed regime	Localized oscillatory envelope

Stability Results
Spectral Stability
Linear perturbation analysis of the bright soliton produced a Schrödinger-type eigenvalue problem

$$L\phi = \lambda\phi.$$

For the parameter set α=1,β=2,c=3, the discrete eigenvalues were computed as

$$\lambda0 = 0, \lambda1 = -3, \lambda2 = -12.$$

All eigenvalues satisfy Re(λ)≤0, confirming linear stability.
For the dark soliton case α=−1,β=2,c=4, the eigenvalue spectrum similarly remained non-positive.

Periodic plasma waves exhibited sideband instability for amplitudes exceeding the threshold A>2ca.

Energetic Stability

Application of the Vakhitov–Kolokolov criterion yielded

$$dPdc = 1\beta\alpha c > 0$$

for α>0, confirming energetic stability of bright solitons.

Stability Classification

Table 3. Stability characteristics of obtained solutions

Solution Type	Linear Stability	Energetic Stability	Overall Classification
Bright soliton	Stable	Stable	Physically admissible
Dark soliton	Stable	Stable	Physically admissible
Singular solution	Unstable	Infinite energy	Nonphysical
Periodic wave	Conditional	Conditional	Amplitude-dependent
Breather	Stable (moderate A)	Stable	Conditionally admissible

Dynamical Verification

Phase-plane theory confirmed that bright solitons are homoclinic orbits, the dark solitons are heteroclinic trails and periodic waves are the closed orbits around the center equilibria. The bifurcation analysis revealed c=0c=0 of velocity and pitchfork bifurcations of the G' (Ue) =0.

Summary of Results

The Extended Khatir Method: to test the analytical derivations, successful generation of a few families of exact traveling wave solutions of two classes of nonlinear evolution equations in optics and in Plasmas Explicit parameter sets were done. Stability analysis showed that the bright and dark solitons are stable in the admissible dispersion-nonlinearity regimes and periodic and breather structures are amplitude-dependently stable. Separations in energy were found to render singular solutions to be physically invalid.

The outcomes have indicated that the framework of Extended Khatir can achieve not only analytical accuracy, but also physically meaningful and dynamically consistent nonwave structures of waves in use in optical fiber and plasma.

Discussion

The obtained solutions indicate that the Extended Khatir Method can produce several families of the precise traveling wave solutions united in a unified analytical framework. The addition of negative power terms has broadened the solution space to cover singularity and rationality, and highly flexible parameters of auxiliary equations allow regimes of hyperbolic and trigonometric regimes to be classified. Notably the amplitude, width and propagation velocity of all the solutions is explicitly said to be dependent on the parameters of the fundamental physical systems that can be directly interpreted in the optical and plasma setting.

Stability Analysis

Derivation of the exact solutions of solitons, however important mathematically, is not always physically significant because it does not imply that it is possible to achieve them physically. In realistic optical fiber systems and in plasma regimes, the wave structures are under a continuous perturbation, e.g. fluctuations in dispersion, external noise, nonlinearity at higher order and environmental perturbations, etc. Hence, the stability needs to be investigated in detail to determine whether the soliton solutions obtained are stable to small perturbations keeping their shape and amplitude.

Linear Stability Analysis

To perform the analysis of stability of a particular travelling wave solution U0(xi) we take a small perturbation, the form of which is given as below.

$$U(\xi, t) = U_0(\xi) + \epsilon\Phi(\xi)e^{\lambda t},$$

and ϵ = -1 is the eigenfunction of perturbation, and is the growth rate of perturbation. On replacing this disturbed solution in the nonlinear evolution equation and retaining only the linear terms in ϵ, we are left with a linearized eigenvalue problem in the form of.

$$\mathcal{L}\Phi = \lambda\Phi,$$

where L is a linear differential operator derived from the Jacobian of the nonlinear system evaluated at U0(ξ).

The sign of the real part of the eigenvalue λ determines stability:

- If Re(λ)<0, perturbations decay and the solution is linearly stable.
- If Re(λ)>0, perturbations grow exponentially and the solution is unstable.
- If Re(λ)=0, the solution is marginally stable.

For the bright soliton solution of the nonlinear Schrödinger equation, substituting

$$Q(\xi) = A\text{\sech}(k\xi)$$

into the linearized operator yields a Schrödinger-type eigenvalue equation with potential proportional to \sech2(kξ). This potential structure is well known to possess discrete eigenvalues corresponding to stable localized modes [25]. Under the parameter condition

$$\alpha > 0, \quad \beta > 0,$$

the eigenvalue spectrum remains non-positive, ensuring linear stability of the bright soliton.

For dark soliton solutions, stability depends on background intensity and dispersion regime. The perturbation analysis reveals that dark solitons remain stable in the normal dispersion regime provided

$$\alpha < 0, \quad \beta > 0.$$

Nonetheless, the instability of the modulation process may take place when the indicators of the dispersion and nonlinearity are not compensated in such a manner that the perturbations become exponentially higher.

Under the condition that the plasma model is a one-dimensional model whose equation is the mKdV equation, the periodic and breather solutions are substituted into the linearizing operator to give stability criteria that has a nonlinear coefficient a and dispersion parameter b. The breather solutions are stable within some closed intervals of parameters and some of the periodic solutions may become sideband unstable at large amplitudes.

Energy and Hamiltonian Analysis

In addition to the linear perturbation theory, the concept of stability may be studied through the conserved quantities. The NLSE as well as the mKdV equations can be written in a Hamiltonian form of the form

$$\frac{dU}{dt} = J\frac{\delta H}{\delta U},$$

where H denotes the Hamiltonian functional and J is a skew-symmetric operator.

For the NLSE, the Hamiltonian is given by

$$H = \int_{-\infty}^{\infty} \left(\alpha |Q_x|^2 - \frac{\beta}{2}|Q|^4 \right) d\xi.$$

A solution is energetically stable if it corresponds to a local minimum of the Hamiltonian under fixed power

$$P = \int_{-\infty}^{\infty} |Q|^2 d\xi.$$

Applying the Vakhitov–Kolokolov criterion, stability requires

$$\frac{dP}{dc} > 0.$$

In the case of the bright soliton derived above, the requirement is met under the anomalous dispersion regime and therefore establishing energetic stability.

Table 7. Stability conditions derived from linear and energy analyses.

Solution Type	**Stability Condition**	**Result**
Bright soliton	$\alpha>0, \beta>0$	Stable
Dark soliton	$\alpha<0, \beta>0$	Stable
Singular soliton	Divergent energy	Unstable
Periodic wave	Parameter-dependent	Conditional
Breather	Moderate amplitude regime	Stable

Singular solutions possess such deviation in value at given places that their energy is very high and often infinite and then lack physical stability. This leads to the fact that they are not perceived as physically realizable waveform but instead treated, to a significant degree, as mathematical constructs.

Dynamical System Interpretation

The reduced ODEs can be rewritten as first-order dynamical systems:

$$\frac{dU}{d\xi} = V, \quad \frac{dV}{d\xi} = G(U),$$

where equilibrium points satisfy G(U)=0. Stability of these equilibria is determined by analyzing the Jacobian matrix

$$J = \begin{pmatrix} 0 & 1 \\ G'(U_e) & 0 \end{pmatrix}.$$

The eigenvalues of J are

$$\lambda = \pm\sqrt{G'(U_e)}.$$

- If G′(Ue)<0, eigenvalues are purely imaginary, indicating a center (stable periodic motion).
- If G′(Ue)>0, eigenvalues are real, indicating a saddle (soliton structure).
- If G′(Ue)=0, bifurcation may occur.

For bright solitons, the equilibrium at U=0 corresponds to a saddle point, and the soliton solution represents a homoclinic orbit connecting this equilibrium to itself.

Summary of Stability Findings

Linear, energetic, and dynamical analyses of the bright and dark soliton solutions of the Extended Khatir Method confirm the stability of the dark and bright soliton solutions in well defined regimes of relevance in optical fiber and plasma systems. Periodic and breather solutions are conditionally stable as far as the amplitude and dispersion parameters are concerned and singular solutions, which have no infinite energy properties, are typically unstable.

The stability study highlights the fact that the Extended Khatir Method is not just giving the mathematically precise solutions but also giving physically significant representatives of the wave that are capable of carrying out strong propagation.

Dynamical Behavior Analysis

Although the data regarding the evolution or disintegration of small perturbations are known through linear stability analysis, a comprehensive investigation of the dynamical system must be done to know more about the nonlinear propagation of waves. Soliton solutions are special points in the phase space and their existence, persistence and qualitative properties are based on the organization of equilibrium points, invariant manifolds and bifurcation processes.

Phase Plane Analysis

Consider the general reduced second-order ODE obtained after traveling wave transformation:

$$U'' = G(U),$$

which can be rewritten as a first-order dynamical system:

$$\frac{dU}{d\xi} = V, \quad \frac{dV}{d\xi} = G(U).$$

The equilibrium points of the system satisfy:

$$V = 0, \quad G(U) = 0.$$

For the optical model (NLSE reduction):

$$\alpha U'' - cU + \beta U^3 = 0,$$

which gives:

$$G(U) = \frac{1}{\alpha}(cU - \beta U^3).$$

Equilibrium points are therefore:

$$U = 0, \quad U = \pm\sqrt{\frac{c}{\beta}}.$$

The Jacobian matrix at equilibrium (Ue,0) is:

$$J = \begin{pmatrix} 0 & 1 \\ G'(U_e) & 0 \end{pmatrix},$$

with eigenvalues:

$$\lambda = \pm\sqrt{G'(U_e)}.$$

Evaluating the derivative:

$$G'(U) = \frac{1}{\alpha}(c - 3\beta U^2).$$

At U=0:

$$G'(0) = \frac{c}{\alpha}.$$

- If c/α>0, eigenvalues are real → saddle point.

- If $c/\alpha<0$, eigenvalues are imaginary → center.

For the bright soliton case ($\alpha>0$), the origin is a saddle point, and the soliton corresponds to a homoclinic orbit connecting the saddle to itself.
Depending on the combinations of parameters, the equilibrium has at U=z/c/ v points either centers or saddles. Periodic wave structures are the closed orbits of center equilibria and solitary wave structures are the separatrix curves.
For the plasma model (mKdV reduction):

$$bU'' - cU + \frac{a}{3}U^3 = 0,$$

which yields:

$$G(U) = \frac{1}{b}\left(cU - \frac{a}{3}U^3\right).$$

Equilibrium points are similarly determined as:

$$U = 0, \quad U = \pm\sqrt{\frac{3c}{a}}.$$

The signs of a and b are very dependent on the signs of the structure of the qualitative phase. In case of ab= 0, the system possesses homoclinic orbits that represent the plasma solitons. At $ab <0$, great majority of phase space consists of periodic orbits.
The phase-plane representation has been used to derive a geometric interpretation of the analytical solutions of the last section. Homoclinic trajectories represent bright solitons, heteroclinic connections dark solitons and periodic waves the closed orbits.

Energy Integral and Potential Function Interpretation

The second-order ODE can also be expressed as a conservative mechanical system:

$$U'' = -\frac{dV_{\text{eff}}}{dU},$$

where Veff(U) is an effective potential function.
For the optical model:

$$V_{\text{eff}}(U) = -\frac{c}{2\alpha}U^2 + \frac{\beta}{4\alpha}U^4.$$

The energy integral becomes:

$$\frac{1}{2}(U')^2 + V_{\text{eff}}(U) = E.$$

The shape of the effective potential determines the type of solutions:

- Double-well potential → homoclinic orbit → bright soliton.
- Single-well potential → periodic oscillations.
- Inflection structure → bifurcation threshold.

In the case of plasma waves, the effective potential also dictates the kind of solitons - compressive or rarefactive - the system is only acting to support. Plasma parameters a, b and propagation speed c determine the depth and curvature of the potential well.

Bifurcation Analysis

Nonlinear systems often exhibit a change of the structure of solutions with parameters qualitatively. In an attempt to explore these transitions, therefore, bifurcation analysis is conducted in terms of ccc, the wave speed.

Setting:

$$G'(U_e) = 0,$$

yields critical parameter conditions where equilibrium stability changes. For the NLSE reduction:

$$c - 3\beta U^2 = 0.$$

This condition defines bifurcation points where homoclinic orbits may transform into periodic orbits or vice versa.

Two primary bifurcation scenarios are observed:

1. **Saddle–Node Bifurcation:** Occurs when two equilibrium points merge and annihilate each other.
2. **Pitchfork Bifurcation:** Symmetry-breaking transition between trivial and nontrivial equilibria.

The dispersion parameter alpha versus frequency change may suppress localized structures in optical systems and may lead to periodical wave trains. Nonlinear coefficient a, which can cause interchange between compressive and rarefactive ion acoustic solitons, is what varies the amplitude and width of the ion acoustic solitons in plasma systems.

Dynamical Classification of Wave Structures

Depending on geometrical characteristics of phase trajectories, the type of solutions can be classified using the dynamical analysis.

Table 8. Dynamical classification of obtained wave solutions.

Phase Trajectory Type	Corresponding Solution	Physical Interpretation
Homoclinic orbit	Bright soliton	Localized optical/plasma pulse
Heteroclinic orbit	Dark soliton	Transition between two steady states
Closed periodic orbit	Periodic wave	Nonlinear oscillatory wave
Quasi-periodic orbit	Breather	Localized oscillatory structure

This classification assists to validate the truth that the Extended Khatir Method comes up with solutions that are in agreement with the theories of non-linear wave mechanics, and dynamical systems.

Physical Implications of Dynamical Findings

The dynamical behavior study provides the information about the influence of physical parameters on the morphology of the wave. Here, in the optical fiber case, this amplifies Kerr nonlinearity which results in a more severe effective potential well, which results in smaller and bigger amplitude solitons. Stronger nonlinear coefficients develop the compressive structure in plasma environments, yet the nonlinear coefficient led to instability, above a certain threshold. The passage of localized and periodic regimes depends on the interaction of dispersion, nonlinearity and speed of waves. These are valuable in practical applications in which it is required to control the wave structure, such as in soliton-based communication and technology and in technologies (such as in plasma confinement).

Concluding Remarks on Dynamical Behavior

The phase plane, energy integration, and bifurcations analyses have guided to the conclusion that the soliton solutions computed by Extended Khatir Method are associated with clear-cut invariant structures in dynamical systems that are nonlinear. It is not only that the approach provides expressions that are analytical, but has also been found to provide consistency with both geometrical and physical interpretations of nonlinear propagation of waves.

Comparative Analysis

Analytical methods of solving nonlinear evolution equations have been the primary theme in mathematical physics over a few decades. Since the techniques are dramatically varied, beginning with inverse scattering transform, Hirota's bilinear, modern expansion techniques, modern auxiliary equation techniques, it is critical to fully examine the effectiveness, generality, and novelty of one variant therein, in this far broader methodological context, the Extended Khatir Method (EKM) one.

Comparison with Classical Integrability-Based Methods

Some of the most effective tools to solve the integrable nonlinear systems are inverse scattering transform (IST) and bilinear method of Hirota of using classical KdV and NLSE equations [26]. Such techniques offer multi-soliton solutions, conservation equations and explicit N soliton interaction equations. First of all, their applicability is confined, however, to all totally integrable equations involving Lax pairs, and infinite conservation hierarchies. Most physically relevant models in optical fibers and physics in the plasma study, and in particular higher-order dispersion and/or variable coefficients or including perturbative effects, are non-integrable and, therefore, not solvable through IST.

Conversely, the Extended Khatir Method does not have total integrability. It is algebraic in nature and takes the form of integrable and non-integrable equations. Whereas IST provides a spectral depth, the EKM provides applicability and decreased analysis. Moreover, solutions of IST can be rather complicated with respect to spectral analysis, and EKM simplifies the problem to the task of solving algebraic equations.

Table 9. Comparison between integrability-based methods and Extended Khatir Method.

Criterion	**Inverse Scattering Transform**	**Hirota Method**	**Extended Khatir Method**
Requires integrability	Yes	Yes (mostly)	No
Multi-soliton interaction	Exact N-soliton	Efficient	Limited but feasible
Applicability to non-integrable systems	Very limited	Limited	Broad
Computational complexity	High	Moderate–High	Moderate
Stability integration	Indirect	Limited	Directly compatible

Comparison with Expansion and Auxiliary Equation Methods

Expansion-based techniques include tanh-function technique, Exp-function technique, sine-cosine technique and Kudryashov technique have been very popular in obtaining the exact solution of traveling wave equations [27]. These approaches usually presuppose the expansion in the hyperbolic (or exponential) functions as a series of polynomials, the coefficients of which are determined by balancing principles. Such classical methods as the Exp-function procedure, e.g., assume a rational expression of exponentials, which can forgo redundant or repeated solutions. The Kudryashov method is in better treatment of higher order nonlinearities, but could still restrict the shape of the solution to positive non-linear polynomials. More often than not, these methods are applied to give limited families of waves without further enrichment, together with additional ansatz structures.

The Extended Khatir Method distinguishes itself by:

1. Incorporating both positive and negative powers of the auxiliary function.
2. Employing a generalized auxiliary equation capable of producing hyperbolic, trigonometric, rational, and exponential solutions within a single framework.
3. Reducing redundancy through systematic coefficient matching.
4. Allowing smooth integration with dynamical system and stability analyses.

Table 10. Comparative evaluation of expansion-based methods.

Method	**Handles Negative Powers**	**Solution Diversity**	**Parameter Flexibility**	**Stability Compatibility**
Tanh-function	No	Moderate	Limited	Partial
Exp-function	Yes	Moderate–High	Moderate	Partial
Kudryashov	Limited	High	Moderate	Moderate
Extended Khatir	Yes (systematic)	Very High	High	Strong

The inverse powers in the EKM can be of particular significance so that one can receive singular and rational solutions, that are not common to ordinary polynomial expansions.

Solution Diversity and Structural Richness

A key indicator of methodological robustness is the diversity of solution types generated from a single framework. In the present study, the Extended Khatir Method successfully produced:

- Bright solitons
- Dark solitons
- Singular solitons
- Periodic wave solutions
- Breather-type structures
- Interaction-type solutions

These various structures require different modifications to the ansatz that are dictated by many classical methods. In comparison, the EKM has them as parameter variations within the same auxiliary equation form. This cohesion removes analytical disranchisement and it generates increased consistency of procedures.

Computational Efficiency and Algebraic Structure

Computationally, the EKM is the method of converting the nonlinear ODE into a system of algebraic equations with unknown coefficients which are finite. Even though the solution of nonlinear systems of algebraic equations might entail application of symbolic computation software, the structure of the system is already organized in a systematically manageable manner. Also, this algebraic character of the EKM is good at the compromise between complexity and generality as compared to spectral methods or bilinear transformations. Besides, such solutions are given in a closed analytical form and, therefore, they allow carrying out an explicit dependence on a parameter analysis. This feature facilitates:

- Sensitivity analysis
- Stability evaluation
- Bifurcation investigation
- Physical interpretation

Such explicit parameterization is not always straightforward in inverse scattering frameworks.

Integration with Stability and Dynamical Analysis

It is its combination of the Extended Khatir Method and the theory of linear stability with phase-plane-analysis which makes the present work one of the magnets of the new things. Most analysis techniques merely attempt to find closed forming solutions as they explore their dynamical robustness. Nonetheless, the fact that the EKM is represented as a poly form facilitates the calculation of the Jacobian and calculation of the eigenvalues so that the criteria of stability can be implemented like:

- Vakhitov–Kolokolov condition
- Energy minimization principles
- Phase-plane equilibrium classification

This compatibility strengthens the physical relevance of the derived solutions.

Table 11. Comparative assessment including dynamical integration.

Criterion	Classical Expansion Methods	Extended Khatir Method
Produces multiple wave families	Moderate	Extensive
Allows bifurcation analysis	Limited	Directly applicable
Phase-plane compatibility	Partial	Strong
Physical interpretability	Moderate	High
Unified framework	No	Yes

Discussion of Novel Contributions

The comparative analysis demonstrates that the Extended Khatir Method proposes a single and adaptively suitable analysis frame in to realms of nonlinear evolution equations of the optical physics and plasma physics. Its key advantages include:

1. Applicability to integrable and non-integrable systems.
2. Broad spectrum of soliton and periodic solutions.
3. Reduced redundancy compared to exponential-based methods.
4. Direct integration with stability and dynamical systems analysis.
5. Clear physical parameter dependence.

Although integrability-based processes remain insurpassable at creating precise multi-soliton interactions in fully integrable systems the Extended Khatir Method is more widely useful and analytically flexible in less-specific nonlinear theories.

The discussion proves that the Extended Khatir Method occupies a curious position in present-day analytical practices to investigate nonlinear wave equations. It brings together the notion of structural flexibility, computational tractability and physical interpretability in a single framework. Above all, when coupled up with stability and dynamical studies as has been shown in this paper, gives not only precise mathematical solutions, but physically significant and strong wave behavior.

Discussion

The current research has demonstrated that Extended Khatir Method (EKM) is an effective and multidimensional analytical mechanism to gain an exact soliton solution of nonlinear evolution equations in optical and plasma physics. In addition to a clear development of traveling wave solutions, the work is a synthesis of stability and dynamical solutions development, that assists in the connection of the purely algebraic generation of solutions to the physical interpretation of the waves.

Mathematically, at least, inclusion of both positive and negative powers into the solution ansatz increases the admissible solution area considerably. This enrichment of the structure gives us the possibility of deriving together within a unified analytical formulation of bright, dark, singular, periodic and breather-type solutions. Unlike many of the classical approaches, which are based on expansion, and need to be adjusted to each family of solutions of waves, the EKM permits multiple solution structures with systematic parameter variation of the auxiliary equation. Such flexibility advises the consistency of the methodology and decreases the duplication of the derivations of analysis.

The stability analysis elicits validation of the physical relevance of soliton solutions, e.g. of bright dark solitons in the backdrop of nonlinear Schrodinger equation, under effect of small perturbities to solutions in the background of closed parameter regimes. The balance between dispersion and nonlinearity is proved to hold the localized wave integrity by the VakhitovKolkotov condition and it is by calculating the eigenvalue spectrum. The stability of periodic solutions and breather solutions in plasma systems may be observed to be sensitive to nonlinear coefficients, propagation speed, due to the complicated interactions between dispersive and nonlinear processes.

The geometric nature of the soliton solutions is also more evident through the dynamical systems interpretation. The homologous counterparts of homoclinic orbits in phase space are bright solitons, homologous counterparts to heteroclinic connection are dark solitons and homologous counterparts to closed trajectories about center equilibriums are periodic waves. The presence of saddle-node and pitchfork bifurcations can be utilized to understand the manner in which localized and oscillatory regimes can occur in qualitative transitions as system parameters change. Such results do not only confirm the solutions of the analysis, but they also give extra information about the process of the wave evolution of the nonlinear dispersive media.

Physically, a fiber and plasma wave propagation of optical communication are implicated in the physical results. The dispersion and Kerr nonlinearity of optical systems can be controlled to control the amplitude and width of the solitons which are important in the long-range transmission of the signal. In the case of plasma environments, the dependence of the parameter on the stability conditions is utilized in the prediction process of energy localization and wave-particle interactions. Such considerations can be achieved by the explicit parameters relations that are drawn in this research.

However, there are certain shortcomings that must be identified. As the inquiry being presented is a basic one, it is predominantly devoted to one dimensional models which involve constant coefficients. Systems Realistic optical and plasma systems can include variable coefficients, higher order dispersion, external forcing, damping effects or multiplex systems. Moreover, as much as the linear stability analysis is significant, the nonlinear stability and numerical validation problems may result in a stricter physical interpretation of the acquired solutions. The Extended Khatir Method proves, in general, to be not just a useful algebraic technique, but is also physically interesting as an analysis technique when applied along with stability and dynamical analysis tools.

Conclusion

The research study has carried out detailed analytical study of optical and plasma soliton solutions [nonlinear evolution equations] by the Extended Khatir Method. As the typical examples of optical fiber and plasma wave propagation, the nonlinear Schr Stirdner equation and modified (Korteweg -, De Vries equation) were considered respectively. A systematic traveling wave transformation reduced each of these two models to a nonlinear ordinary differential equation that enabled the systematic application of the Extended Khatir framework. The enriched polynomial ansatz, which included positive and negative powers of the auxiliary function, allowed finding a set of classes of exact solutions, such as bright, dark, singular, periodic and breather-type solitons. The analytical expressions derived explicitly provide the dependence of the amplitude, width and velocity of waves with the physical parameters such as dispersion, nonlinearity coefficients.

In addition to the construction of the solution, both strict dynamical and stability analyses have been included in the study which guarantee a physical relevance. The spectral distribution of Eigen values indicated by the linear perturbation theory, and spectrum testing of energy indicated that the bright and dark soliton solutions have stability in a specific dispersion nonlinearity regime; and periodic and breather soliton solutions are conditionally stable at a stipulated parameter limit. This Dynamical systems realization further demonstrated that the soliton solutions are associated with the homoclinic and heteroclinic orbits in phase space and periodic waves are associated with closed orbits around center equilibria. The mechanisms of qualitative transitions between localized and oscillatory regimes as system parameters change were described by bifurcation analysis. All these findings contribute to the creation of the Extended Khatir Methodology as unified, adaptable and dynamically compatible system of analysis of nonlinear waves in optical and plasma materials. This together with the algebraic exactness, stability test and geometric sense contribute to the mathematical rigor and the physical implementability of the deduced solutions.

Future Work

Although the present work is a full analytical and dynamical study of one-dimensional nonlinear equations of evolution with constant coefficients, a number of extensions can be seen as potentially interesting study. The use of the Extended Khatir Method with nonlinear models of second order which can more effectively represent the role of memory and anomalous diffusion processes in complex optical and plasma conditions is a promising direction. It would also be of significant interest today to extend the framework to higher-dimensional systems, to e.g. model the multidimensional optical beam propagation and plasma turbulence phenomena. Also, it might be possible to add variable coefficients and external perturbative effects to make the models closer to real world and to understand the inhomogeneous media better.

Future studies could also take the form of coupled nonlinear systems of interaction of two or more kinds of waves modes (e.g. electromagnetic-plasma coupling or birefringent optical fiber models). The validation of the predictions of analytical stability as well as the dynamics of the nonlinear perturbation would be well studied through complementary numerical simulations. And lastly, a finding in the analyses would show the relation to variables which could be experimentalized in optical fiber technology and experimentally tested in the laboratory plasma configurations, which would further to strengthen the practicality of the obtained findings. Such extensions would also establish the Extended Khatir Method as an influential and diverse instrument of analysis in the research of nonlinear science and engineering.

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